FM Stereo Receiver

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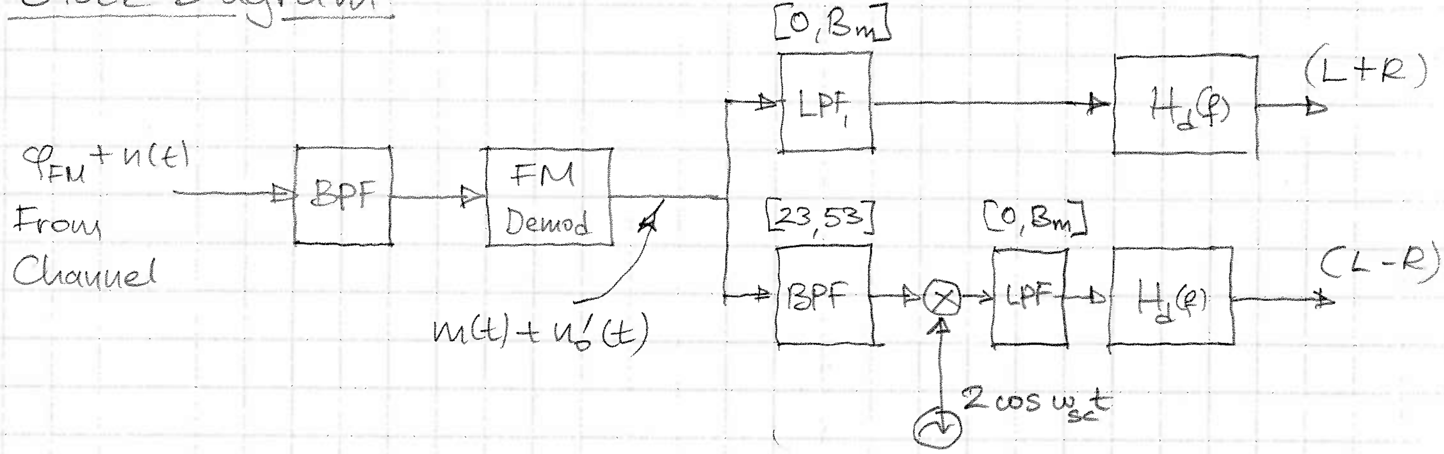
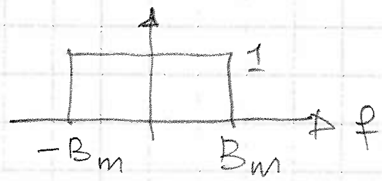
For FM stereophonic broadcasting show that the (L−R) channel is about 22 dB noisier than the (L+R) channel.

Block Diagram

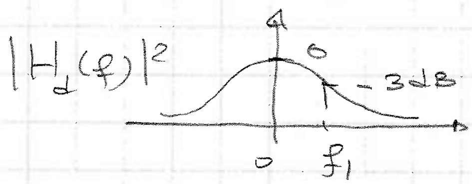


with $LPF_1 = LPF_2$



$B_m = 15\,kHz$

$H_d(f)$ = de-emphasis filter



$f_1 = 2.1\,kHz \quad RC = 75\mu s.$

$$|H_d(f)|^2 = \frac{f_1^2}{f_1^2 + f^2}$$

Output of FM demodulator

$$m(t) = (m_L + m_R) + (m_L - m_R)\cos\omega_{sc}t + \cos\frac{\omega_{sc}}{2}t + n_o'(t)$$

with $(m_L + m_R)$ mono signal band limited to B_m

$(m_L - m_R)$ stereo signal band limited to B_m

$f_{sc} = 38\,kHz$ subcarrier for $(m_L - m_R)$

$f_{sc}/2 = 19\,kHz$ pilot tone for synch. detection of $m_L - m_R$

$n_o'(t)$: noise at the output of the FM demodulator with

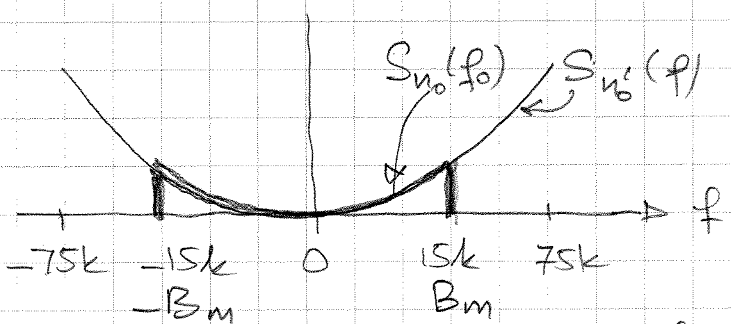
$$S_{n_o'}(f) = \frac{N}{A_c^2}(2\pi f)^2 \quad |f| \le 75\text{ kHz}^*$$

* bandwidth of $m(t)$ including (m_L+m_R), $(m_L-m_R)\cos\omega_{sc}t$ and and SCMO/SCA signal.

Noise in Mono signal

(a) Without De-emphasis.

$$n_o'(t) \longrightarrow \boxed{\text{LPF}} \longrightarrow n_o$$



$$S_{n_o}(f) = |H_{LP}(f)|^2 S_{n_o'}(f) = \begin{cases} \frac{N}{A_c^2}(2\pi f)^2, & |f| \le B_m \\ 0, & \text{otherwise} \end{cases}$$

$$N_o = \overline{n_o^2} = \int_{-B_m}^{B_m} S_{n_o}(f)\,df = \frac{4\pi^2 N}{A_c^2}\int_{-B_m}^{B_m} f^2\,df$$

$$= \frac{8\pi^2 N}{A_c^2}\int_0^{B_m} f^2\,df = \frac{8\pi^2 N}{3A_c^2}\, f^3\Big|_0^{B_m}$$

$$\boxed{N_o = \frac{8\pi^2 N B_m^3}{3A_c}\ [W]}$$

(b) With De-emphasis

$$n_o' \longrightarrow \boxed{\text{LPF}} \longrightarrow \boxed{H_d(f)} \longrightarrow n_{o-d}$$

$$S_{n_o}(f) = \begin{cases} |H_d(f)|^2 S_{n_o'}(f), & |f| \le B_m \\ 0, & \text{otherwise} \end{cases}$$

$$N_{o-d} = \overline{n_{o-d}^2} = \frac{4\pi^2 N}{A_c^2} \int_{-B_m}^{B_m} f_1^2 \frac{f^2}{f_1^2 + f^2}\, df$$

Using..

$$\int \frac{x^2}{a^2+x^2}\,dx = x - a\tan^{-1}\left(\frac{x}{a}\right)$$

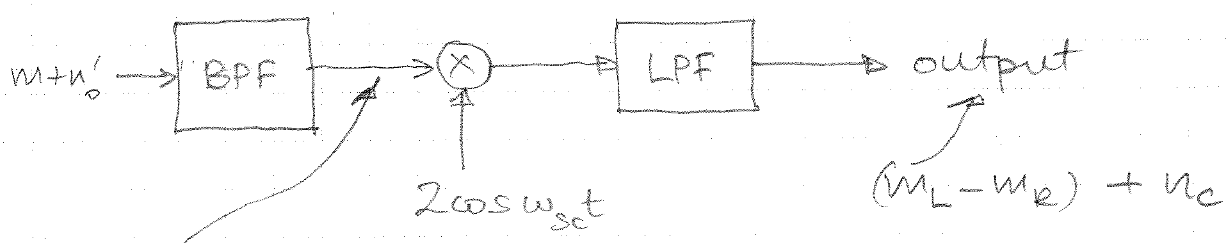
$$= \frac{8\pi^2 N f_1^2}{A_c^2} \int_0^{B_m} \frac{f^2}{f_1^2 + f^2}\, df$$

$$= \frac{8\pi^2 N f_1^2}{A_c^2} \left[f - f_1 \tan^{-1}\left(\frac{f}{f_1}\right) \right]_0^{B_m}$$

$$\boxed{N_{o-d} = \frac{8\pi^2 N f_1^2}{A_c^2} \left[B_m - f_1 \tan^{-1}\left(\frac{B_m}{f_1}\right) \right]}$$

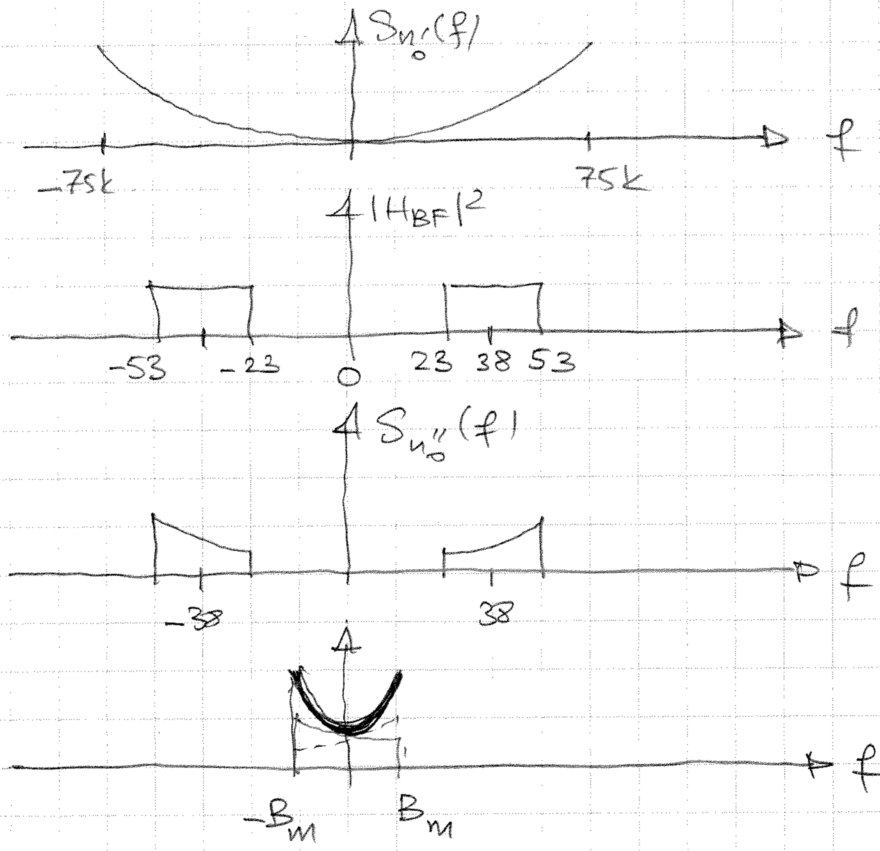
Noise in Stereo Signal

(a) Without De-emphasis.



$$(m_L - m_R)\cos w_{sc}t + \underbrace{\left[n_c \cos w_{sc}t + n_s \sin w_{sc}t \right]}_{n_o''}$$

with



$S_{n_c}(f) = S_{n_s}(f)$

$$S_{n_c}(f) = S_{n_s}(f) = S_{n_0''}(f - f_{sc}) + S_{n_0''}(f + f_{sc}), \ |f| \le B_M$$

$$= S_{n_0'}(f - f_{sc}) + S_{n_0'}(f + f_{sc})$$

$$= \frac{4\pi^2 N}{A_c^2}\left[(f - f_{sc})^2 + (f + f_{sc})^2\right]$$

$$= \frac{4\pi^2 N}{A_c^2}\left[2f^2 + 2f_{sc}^2\right]$$

$$S_{n_c}(f) = \frac{8\pi^2 N}{A_c^2}\left[f^2 + f_{sc}^2\right], \quad |f| \le B_M$$

$$N_{0\text{-}st} = \overline{n_c^2} = \int_{-B_M}^{B_M} \frac{8\pi^2 N}{A_c^2}\left[f^2 + f_{sc}^2\right] df$$

$$= \frac{16\pi^2 N}{A_c^2} \int_0^{B_M} \left[f^2 + f_{sc}^2\right] df$$

$$= \frac{16\pi^2 N}{A_c^2}\left[\frac{f^3}{3}\Big|_0^{B_m} + f_{sc}^2\, f\Big|_0^{B_m}\right]$$

$$\boxed{N_{o\text{-}st} = \frac{16\pi^2 N B_m}{A_c^2}\left[\frac{B_m^2}{3} + f_{sc}^2\right]}$$

(b) With De-emphasis

$$n_c \longrightarrow \boxed{H_d} \longrightarrow n_{o\text{-}d}$$

$$S_{n_o}(f) = |H_d(f)|^2 S_{n_c}(f)$$

$$= \frac{8\pi^2 N}{A_c^2}\left[f^2 + f_{sc}^2\right]\left[\frac{f_1^2}{f_1^2 + f^2}\right], \quad |f| \le B_m$$

$$N_{o\text{-}st\text{-}d} = \overline{n_{o\text{-}d}^2} = \frac{8\pi^2 N}{A_c^2}\int_{-B_m}^{B_m}\left[f_1^2\,\frac{f^2}{f_1^2 + f^2} + f_{sc}^2 f_1^2\,\frac{1}{f_1^2 + f^2}\right] df$$

$$= \frac{16\pi^2 N}{A_c^2} f_1^2\left[\int_0^{B_m}\frac{f^2}{f_1^2 + f^2}\,df + f_{sc}^2\int_0^{B_m}\frac{df}{f_1^2 + f^2}\right]$$

$$\boxed{N_{o\text{-}st\text{-}d} = \frac{16\pi^2 N f_1^2}{A_c^2}\left[B_m + \frac{f_{sc}^2 - f_1^2}{f_1}\tan^{-1}\left(\frac{B_m}{f_1}\right)\right]}$$

Using $\displaystyle\int \frac{x^2}{a^2 + x^2}\,dx = x - a\tan^{-1}\left(\frac{x}{a}\right)$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

Summary

Output Noise	Without De-emphasis	With De-emphasis
Mono Signal $(m_L + m_R)$	N_{o-m}	N_{o-m-d}
Stereo Signal $(m_L - m_R)$	N_{o-st}	N_{o-st-d}

with

$$N_{o-m} = \frac{8\pi^2 \mathcal{N} B_m^3}{3A_c^2}$$

$$N_{o-m-d} = \frac{8\pi^2 \mathcal{N} f_1^2}{A_c^2}\left[B_m - f_1 \tan^{-1}\left(\frac{B_m}{f_1}\right)\right]$$

$$N_{o-st} = \frac{16\pi^2 \mathcal{N} B_m}{A_c^2}\left[\frac{B_m^2}{3} + f_{sc}^2\right]$$

$$N_{o-st-d} = \frac{16\pi^2 \mathcal{N} f_1^2}{A_c^2}\left[B_m + \frac{f_{sc}^2 - f_1^2}{f_1}\tan^{-1}\left(\frac{B_m}{f_1}\right)\right]$$

<u>Comparison</u>

$$\frac{N_{o-st}}{N_{o-m}} = \frac{\frac{16\pi^2 \mathcal{N} B_m}{A_c^2}\left[\frac{B_m^2}{3} + f_{sc}^2\right]}{\frac{8\pi^2 \mathcal{N} B_m^3}{3A_c^2}}$$

$$= 2\,\frac{3}{B_m^2}\left[\frac{B_m^2}{3} + f_{sc}^2\right]$$

$$= 2\left[1 + 3\,\frac{f_{sc}^2}{B_m}\right]$$

$$\boxed{\frac{N_{o-st}}{N_{o-m}} = 2\left[1 + 3\,\frac{(38000)^2}{(15000)^2}\right] = 40.5 \Longrightarrow 16.1\ \text{dB}}$$

$$\frac{N_{o\text{-}st\text{-}d}}{N_{o\text{-}m\text{-}d}} = \frac{\dfrac{16\pi^2 N f_1^2}{A_c^2}\left[B_m + \dfrac{f_{sc}^2 - f_1^2}{f_1}\tan^{-1}\left(\dfrac{B_m}{f_1}\right)\right]}{\dfrac{8\pi^2 N f_1^2}{A_c^2}\left[B_m - f_1\tan^{-1}\left(\dfrac{B_m}{f_1}\right)\right]}$$

$$= 2\,\frac{\left[B_m + \dfrac{f_{sc}^2 - f_1^2}{f_1}\tan^{-1}\left(\dfrac{B_m}{f_1}\right)\right]}{\left[B_m - f_1\tan^{-1}\left(\dfrac{B_m}{f_1}\right)\right]}$$

Using the numeric values $B_m = 15$ kHz, $f_{sc} = 38$ kHz and $f_1 = 2.1$ kHz, we have

$$\frac{N_{o\text{-}st\text{-}d}}{N_{o\text{-}m\text{-}d}} = 2\,\frac{\left[15000 + \dfrac{(38000)^2 - (2100)^2}{2100}\tan^{-1}\left(\dfrac{15000}{2100}\right)\right]}{\left[15000 - 2100\tan^{-1}\left(\dfrac{15000}{2100}\right)\right]}$$

$$\boxed{\frac{N_{o\text{-}st\text{-}d}}{N_{o\text{-}m\text{-}d}} = 166.2 \Longrightarrow 22.2\text{ dB}}$$